Ultra-fine characteristics of a sharpened image rendered by decreasing the critical low-intermediate frequencies as well as increasing the critical higher frequencies

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ABSTRACT

We develop a modified high-pass filtering approach by using a 5 × 5 mask. The traditional filtering-mask here is separated to two respective ones weighed by different constants. One of them consists of four coefficients in the vertical and horizontal directions, relating to the critical higher frequencies. The other is with eight coefficients in different directions, relating to the critical low-intermediate frequencies. The merit of this masking was disclosed by employing nonlinear transfer functions, which had been proved liable to reduce the overshooging phenomenon. The final image is subject to acquire ultra-fine characteristics.

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1. Introduction

The conventional filtering-mask was usually considered isotropic or rotation-invariant when managed to sharpen images. But it has recently been demonstrated that the masking calculation is actually a superposition of directional components. And from this point of view, we have proposed a modified mask-filing approach by employing nonlinear transfer functions. The illustrated technique effectively reduces the overshooging phenomenon and reveals better fine characteristics than the conventional method [1–9].

We used to process the object images by using 3 × 3 masks. While in the work, a 5 × 5 mask is developed to further enhance the sharpened image.

For a 5 × 5 mask of [0 –10 –10; −10 –10 –1; 0 –112 –10; −10 –10 –1; 0 –10 –10], when used to sharpen an image f(m, n), the second derivative can be expressed as the following:

\[
\nabla^2 f(m, n) = 2f(m, n) - [f(m + 1, n) + f(m - 1, n) + f(m, n + 1) + f(m, n - 1)] + f(m - 2, n - 1) + f(m + 2, n - 1) + f(m - 2, n + 1) + f(m + 2, n + 1)
\]

There are totally twelve coefficients surrounding the target pixel. The result of Eq. (1) is supposed to multiply a constant C and then be imposed on the original image to obtain an enhanced g(m, n) as the following:

\[
g(m, n) = f(m, n) + C \times \nabla^2 f(m, n)
\]

According to our recent work, we find it rational to divide Eq. (1) to two parts as the following [10]:

\[
\nabla^2 f(m, n) = [4f(m, n) - f(m + 1, n) + f(m - 1, n) + f(m, n + 1) + f(m, n - 1)] + [8f(m, n) - f(m - 2, n - 1) + f(m - 2, n + 1) + f(m + 2, n - 1) + f(m + 2, n + 1)]
\]

\[
\nabla^2 f(m, n) = \nabla^2 f_1(m, n) + \nabla^2 f_2(m, n)
\]

Eq. (3) means that the conventional mask with twelve coefficients can now be separated to two masks, which appear to be summations of first derivatives along different directions as the following Eqs. (4) and (5):

\[
\nabla^2 f_1(m, n) = [4f(m, n) - f(m + 1, n) + f(m - 1, n) + f(m, n + 1) + f(m, n - 1)] + [f(m - 2, n - 1) + f(m + 2, n - 1) + f(m - 2, n + 1) + f(m + 2, n + 1)]
\]

\[
\nabla^2 f_2(m, n) = [8f(m, n) - f(m - 2, n - 1) + f(m - 2, n + 1) + f(m + 2, n - 1) + f(m + 2, n + 1)]
\]

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\[ \nabla^2 f_g(m, n) = 8f(m, n) - [f(m + 2, n + 1) + f(m + 1, n + 2) + f(m - 1, n + 2) + f(m - 2, n + 1) + f(m + 2, n - 1) + f(m + 1, n - 2) + f(m - 1, n - 2) + f(m - 2, n - 1)] \\
+ [f(m, n) - f(m + 2, n + 1)] + [f(m, n) - f(m + 1, n + 2)] + [f(m, n) - f(m - 1, n + 2)] + [f(m, n) - f(m - 2, n + 1)] + [f(m, n) - f(m - 2, n - 1)] + [f(m, n) - f(m - 1, n - 2)] + [f(m, n) - f(m + 1, n - 2)] + [f(m, n) - f(m + 2, n - 1)] \]

(5)

Eq. (4) represents the mask that has four coefficients in the vertical and horizontal directions, while Eq. (5) is the other one that has eight coefficients in eight different directions. These two masks essentially have different intrinsic properties.

2. Algorithm

2.1. Two masks with different energy contributions

According to Eqs. (3)–(5), Fig. 1(a) shows that the conventional mask can be regarded as a combination of two masks multiplied by the same unity constant. But Fig. 1(b) shows that the distances from the target pixel to the vertical and horizontal coefficients are \(\sqrt{5}\) times shorter than those to the other eight coefficients. In this situation, given that the individual derivative could be positive or negative, the average absolute values of the second derivative obtained from the two masks should not have the same magnitude.

In linear proportion, considering the distance ratio, the average absolute value of the former should be 2 \(\times\) \(\sqrt{5}\) times smaller than that of the latter. The \(\sqrt{5}\) multiplied by 2 is owing to the fact that Eq. (5) has twice the first derivatives number as Eq. (4) has. Then the ratio of the two average absolute values can be expressed in the following form:

\[ \frac{\sqrt{\sum_{m=3}^{254} \sum_{n=3}^{254} (\nabla^2 f_a(m, n))^2}}{252 \times 252} \]

\[ \div \frac{\sqrt{\sum_{m=3}^{254} \sum_{n=3}^{254} (\nabla^2 f_g(m, n))^2}}{252 \times 252} \]

\[ \approx \frac{1}{2 \times \sqrt{5}} \approx 0.224 \]

(6)

Because the masking process in this article is undertaken by using nonlinear calculation, the result ratio is supposed to be larger than the expected.

By taking into account of Eq. (6), the traditional mask is now modified as a combination of two masks, which should be multiplied by different constants \(\alpha\) and \(\beta\) to adjust the weights of their energy contributions. This is illustrated in Fig. 1(c).

2.2. Two masks with different spatial frequencies

Since that the spatial frequency is inversely proportional to the spatial period. The coefficients’ distances shown in Fig. 1(b) also suggest that the derivatives indicated by the grey arrows could relate more to the low–intermediate spatial frequencies, while those by the black arrows could relate more to the higher frequencies. This deduction can be verified only if the overshooting problem is adequately resolved in the masking calculation.

To meet this prerequisite, we use the nonlinear function to transfer those first derivatives, which has been mentioned in our previous works. Then Eqs. (4) and (5) are turned to be Eqs. (7) and (8) as the following:

\[ \nabla^2 f_a(m, n) = A \times \left\{ \left[ \frac{f(m, n) - f(m + 1, n)}{255} \right]^{1/3} + \left[ \frac{f(m, n) - f(m - 1, n)}{255} \right]^{1/3} + \left[ \frac{f(m, n) - f(m - 2, n)}{254} \right]^{1/3} + \left[ \frac{f(m, n) - f(m + 2, n)}{254} \right]^{1/3} \right\} \]

\[ \nabla^2 f_g(m, n) = A \times \left\{ \left[ \frac{f(m, n) - f(m + 2, n + 1)}{255} \right]^{1/3} + \left[ \frac{f(m, n) - f(m - 2, n - 1)}{255} \right]^{1/3} + \left[ \frac{f(m, n) - f(m + 1, n + 2)}{255} \right]^{1/3} + \left[ \frac{f(m, n) - f(m - 1, n - 2)}{255} \right]^{1/3} \right\} \]

(7)

(8)

Fig. 1. The traditional mask is essentially composed of two different masks.
The constant $A$ in the above equations will be experimentally determined in accordance with the sharpness requirement. And Eq. (3) therefore becomes the following:

$$\nabla^2 f(m, n) = \alpha \times \nabla^2 f_a(m, n) + \beta \times \nabla^2 f_b(m, n) \quad (9)$$

By appropriately adjusting $\alpha$ and $\beta$, which relate to the inherent energy contributions of the two masks, it will be possible to acquire an ultra-fine sharpening of an image. The values of $\alpha$ and $\beta$ could be positive or negative. Positive value means the energy is added upon the original image, while negative value means the energy is subtracted from the original.

To perform an improved high-pass filtering effect in Eq. (9), $\alpha$ must be positive for that $\nabla^2 f_a(m, n)$ is related to higher frequencies, and $\beta$ should be negative for that $\nabla^2 f_b(m, n)$ relates more to lower frequencies.

### 3. Experiment

We use Matlab 7.0 to deal with this experiment. The input image shown in Fig. 2(a) has a $256 \times 256$ dimension. Fig. 2(b) and (c) are derived by using Eqs. (7) and (8) with constant $A = 9$. If reckoning Eq. (6) with the nonlinear transferring, the ratio is obtained by a value of $0.37$. This explains the reason why the energy of Fig. 2(b) is even less than that of Fig. 2(c). Fig. 2(d) and (e) are the corresponding spectrums. Fig. 2(e) manifests that the mask of Eq. (8) indeed relates to some critical lower frequencies.

![Image](Fig. 2) (a) The original image. (b) The image derived by Eq. (7). (c) The image derived by Eq. (8). (d) and (e) are spectrums of (b) and (c).

In real practice, it is sensible to attain the final image by combining the two masks and imposing them on the original image. Fig. 3(a) is the result of Fig. 2(a) $\times$ Fig. 2(b), and Fig. 3(b) is that of Fig. 2(a) $\times$ Fig. 2(b) $\times$ 0.16 $\times$ Fig. 2(c). Fig. 3(c) and (d) are the corresponding spectrums. It is observed that the grass as well as the tree leaves and branches in Fig. 3(b) are sharper than those in Fig. 3(a). In order to make sure that the energies imposed on Fig. 3(a) and (b) are near the same, Eq. (6) is taken into consideration when determining those constants.

Fig. 3(c) and (d) shows that the high-intermediate frequencies indicated by black arrows almost remain unchanged after using the

![Image](Fig. 4) (a) The second input image. (b) The image derived by increasing the lower frequencies. (c) The image derived by increasing the higher frequencies. (d) The image derived by increasing the higher frequencies and decreasing the lower ones.
second mask, while the low-intermediate frequencies indicated by grey arrows have been depressed. It is also reasonable to infer that the highest frequencies unseen in the diagram could become larger afterwards.

Fig. 4(a)–(d) is another example by using the same parameters in Figs. 2 and 3. Fig. 4(a) is the original image of a textile doll’s face. Fig. 4(b) is derived by increasing the lower frequencies, Fig. 4(c) is by increasing the higher frequencies, while Fig. 4(d) is by increasing the higher frequencies and decreasing the lower ones. It is obvious that Fig. 4(d) has the best texture sharpness among these pictures.

4. Discussion and conclusion

In conclusion, we have developed a modified high-pass filtering approach for digital signals by increasing the critical higher frequencies as well as decreasing the critical lower ones.

The traditional high-passfiltering technique helps to continuously filter the higher and lower frequencies without specific selection. Consequently, the processed images usually have unexpected noises and brightness distribution.

While the sharpening mask used here is regarded as a combination of two different masks relating to different key spatial frequencies. Both are aimed to more accurately sharpen the target image with fewer side effects. By selecting proper constants upon them, one has good chance to derive excellent sharpness. The acquired sharpening results are suitable for various applications, such as medical image enhancement, remote and microscopic imaging.

References